Non-Linear Stability of an Electrified Plane Interface in Porous Media

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The non-linear electrohydrodynamic stability of capillary-gravity waves on the interface between two semi-infinite dielectric fluids is investigated. The system is stressed by a vertical electric field in the presence of surface charges. The work examines a few representative porous media configurations. The analysis includes Rayleigh-Taylor and Kelvin-Helmholtz instabilities. The boundary – value problem leads to a non-linear equation governing the surface evolution. Taylor theory is adopted to expand this equation, in the light of multiple scales, in order to obtain a non-linear Schrödinger equation describing the behavior of the perturbed interface. The latter equation, representing the amplitude of the quasi-monochromatic traveling wave, is used to describe the stability criteria. These criteria are discussed both analytically and numerically. In order to identifiy regions of stability and instability, the electric field intensity is plotted versus the wave number. Through a linear stability approach it is found that Darcy's coefficients have a destabilizing influence, while in the non-linear scope these coefficients as well as the electric field intensity play a dual role on the stability.

Key words: Non-linear Stability; Porous Media; Electrified Plane Interface; Multiple Scale Technic.

1. Introduction

The Rayleigh-Taylor instability (RTI) occurs when a heavy fluid is supported by a lighter one in a gravitational field, or equivalently, when a heavy fluid is accelerated by a lighter one. The RTI has been addressed in several studies because of its importance for stratified systems, among which planatary and stellar atmospheres are examples. The effect of external forces is mainly important in planetary and stellar systems. Coriolis and centrifugal forces are common in these systems and play an important role in determining many phenomena including the RTI. Different properties of fluids have been included in the RTI through theoretical investigations. For instant, Chandrasekhar [1] included the viscosity and Reid [2] the effects of both viscosity and surface tension. The study of the non-linear RTI has received a considerable number of contributions. Nayfeh [3] studied the stability of a surface wave propagating through a horizontal interface between two inviscid fluids and derived a non-linear Schrödinger equation describing the disturbance. The stability of the system could be judged by the sign of the coefficients of this equation. Moreover, Nayfeh used the non-linear Schrödinger equation to obtain the cutoff wave number separating stable and unstable disturbances. On the other hand, Oron and Rosenau [4] studied the non-linear evolution of interfacial waves separating two liquids of different viscosity and density in two-dimensional channel. They found that the non-linear evolution of the interface is governed by the regularized Kuramoto-Sivashinsky equation. El-Dib [5] studied the non-linear hydromagnetic RTI for strong viscous fluids in porous media. He derived stability criteria and have been performed for non-linear Chandrasekhar dispersion relation including porous effects.

The KHI occurs when two fluids are in a relative motion on either side of a common interface. At the same time, the KHI can also develop in cases when there is no initial velocity jump, but the system dynamics is such that separate induced flows appear during the evolution of the system. Because of its relevance to a variety of space and astrophysical phenomena involving sheared plasma flow (e.g., the stability of the solar-wind-magneto-sphere interface, interaction between adjacent streams of different velocities in the solar wind and dynamo generation of cosmic magnetism) in geophysical and laboratory situations, this problem has been analyzed by several authors [6]. Without surface tension, this streaming is unstable no matter how small the velocity difference between the two layers may be. It was early shown by Kelvin [7] that the surface tension will suppress the instability if the difference in velocity is sufficiently small. A detailed account of the instability of the plane interface between two Newtonian fluids, under varying assumptions of hydrodynamics and hydromagnetics, has been given by Chandrasekhar [8]. He has analyzed and discussed in details the effect of surface tension, variable density, streaming velocity, rotation and magnetic field on the KHI. For an excellent review see Drazin and Ried [9]. Malik and Singh [10] investigated the non-linear stability of such a configuration. They demonstrated that for subcritical flows, i. e. when the velocity difference is less than the critical velocity, the evolution of the amplitude is governed by a non-linear Schrödinger equation. They showed that the wave train of constant amplitude is unstable against modulations. Furthermore, in the subscritical regime, there are two stable and two unstable regions. Weissman [11] examined the problem of the KHI near the critical point in the parameter space and for regions of linear stability. El-Dib [12] investigated the effect of dielectric viscoelastic interface on the non-linear KHI. His numerical calculaions showed that the difference between the vertical fields produces a destabilizing influence in the stability criteria. Moatimid [13] studied the non-linear KHI of two miscible ferrofluids in porous media. Through a non-linear approach, he obtained a Ginzburg – Landau equation describing the surface evolution. He discussed the stability criteria in bot theoretically and computationally.

Electrohydrodynamics (EHD) can be regarded as the branch of fluid mechanics concerned with electrical-force effects, or as the part of electrodynamics that deals the influence of moving media in electric fields. It is thus concerned with the interaction between electric fields and free or polarized charges in fluids. The fluids may be extremely good insulators, slightly conducting, or even highly conducting. Although the literature in the field of interfacial fluid flows is vast, a very small portion of it has been devoted to the investigation of surface wave phenomena in EHD. A survey on EHD with special reference to many of the developments in this field is given by Melcher [14]. In the linear EHD stability theory it is known that the tangential electric field has a stabilizing effect, while the normal one always has a destabilizing influence. Applications of EHD include pumping and levitation of liquids and gases, extraction of contaminants from gases such as smoke, mixing of liquids and orientation of liquids in/near-zero-gravity environments. EHD interactions also occur in meteorology, in which the charge distribution in the atmosphere is important, and in surface physics, in which the distribution of charges at interfaces is significant for frictional electrification. The EHD instability of a plane interface stressed initially by a perpendicular electric field has been studied by Moatimid [15], Melcher and Smith [16], and others. With non-linear EHD KHI deals a considerable number of contributions, e. g. by Mohamed and Elshehawey [17]. They studied the stability of a horizontal interface, separating two dielectric streaming fluids stressed by a perpendicular electric field. They obtained two non-linear Schrödinger equations. In their analysis they found that both the streaming and electric field play a role on the stability criterion. Mohamed *et al.* [18] investigated the non-linear gravitational stability of streaming in an electrified viscous flow through porous media. They solved the linear equations of motion in the light of the non-linear boundary conditions. El-Dib [19] studied the non-linear electrorheological instability of two Rivlin – Ericksen elastico – viscous fluids. The perturbation analysis, in the light of the multiple time scales, leads to imposing the well known non-linear Schrödinger equation having complex coefficients. His numerical calculations showed that the ratio of the dielectric constants plays a dual role in the stability crteria. Moatimid [20] investigated the non-linear electrorheological instability of two streaming cylindrical fluids. The stability criteria are discussed theoretically and illustrated graphically in which the stability diagrams are obtained.

Many technological processes involve the parallel flow of fluids of different viscosity and density through porous media. Such parallel flows exist in packed-bed reactors in the chemical industry, in petroleum-production engineering, in boiling in porous media (countercurrent flow of liquid and vapor), and in many other processes. There are two reasons to extend the previous studies in non porous media, to media containing porous layers: (i) the KHI is among the most simple and (ii) among motions in the presence of a porous medium the KHI is the most frequently met in practice. Flow through a porous medium has been of considerable interest in recent years particular among geophysical fluid dynamics. For excellent reviews about porous media see [21 – 23]. The instability of the plane interface between two uniform superposed streaming fluids through porous media has been investigated by many authors for different cases of interest. It was shown by Sharma and Spanos [24] that the surface tension is able to suppress the KHI for small wave length perturbations, and the medium porosity reduces the stability range given in terms of a difference in streaming velocities. They also found that for the top-heavy configurations, the surface tension stabilizes a certain wave number range. It was pointed out by Bau [25] that, when the fluids are moving parallel to each other at different velocities, the interface may become unstable. He has extended the KHI for flow in porous media for the cases of Darcian and non-Darcian flows. El-Dib and Ghaly [26] studied the non-linear interfacial stability for magnetic fluids in porous media. Through the non-linear scope, they found that the increase of the resistance parameters plays both stabilizing and destabilizing role in the stability criteria. The non-linear waves on the surface of a magnetic fluid jet in porous media is studied by Moatimid [27]. He found that the porous media have a destabilizing influence. This influence is enhanced when the Darcy's coefficients are different. Also, new instability regions in the parameter space, which appear due to the non-linear effects are shown. Recently, Moatimid and El-Dib [28] studied the non-linear KHI of Oldroydian viscoelastic fluid in porous media. Their analysis resulted in a non-linear Ginzburg – Landau equation, describing the competition between the non-linearity and dispersion.

From a geophysical standpoint, the onset instability in dielectric fluids under the action of an applied electric field in porous media is of great interest. Therefore we have considered a simplified mathematical formulation of the problem of interfacial flow of the non-linear RTI as well as the KHI types, use the stability configuration of the perturbed system composed of two incompressible, homogeneous dielectric and streaming fluids separated by a horizontal interface under the action of a normal electric field. The work examines a few representative porous media configurations. This study is assumed to be more suitable for problems in the oil industry. In the following, we shall first formulate the general interfacial problem for two superposed dielectric fluids in the presence of a normal electric field producing surface charges. Then the non-linear analysis, using a multiple scale expansion is carried out. The stability criteria are discussed both analytically and numerically, and the stability diagrams are drawn.

2. The Darcian Flow and Exposition of the Problem

The system under consideration consists of two homogenous incompressible fluids. Initially the interface between the fluids forms the plane $y = 0$. $\rho^{(1)}$ and $\rho^{(2)}$ are the densities, and $\varepsilon^{(1)}$ and $\varepsilon^{(2)}$ the dielectric constants of the upper and lower fluid, respectively. Gravity acts in the negative y-direction. The fluids are stressed by the electric fields $E_0^{(1)}$ and $E_0^{(2)}$, acting in the negative y-direction. There are surface charges on the interface, i.e. $\varepsilon^{(1)}E_0^{(1)} \neq \varepsilon^{(2)}E_0^{(2)}$. Thus we have

$$\boldsymbol{E}_0^{(j)} = -E_0^{(j)}\boldsymbol{e}_y, \qquad j = 1,2, \tag{2.1}$$

where $\boldsymbol{e}_y$ is the unit vector in the y-direction.

At the initial state of the system, we assume that both the fluid phases are immiscible and have a common flat interface at $y = 0$. The equilibrium distribution of the interface between both fluid phases has been established. Attention is focused to the interfacial response of the two fluids after disturbance of the equilibrium configuration. The interface between the two fluids is physically a surface in three dimensions that is always composed of the same particles, without any loss of generality, all disturbances are assumed to be two dimensional. By two-dimensional we mean that the flow depends on the horizontal x-direction of the propagation, i. e.

$$y = \eta(x,t). \tag{2.2}$$

The particles will always remain on a surface defined by $S(x,y,t) = 0$. Electromagnetic boundary conditions are commonly formulated in terms of a unit-normal vector $\boldsymbol{n}$, Because this vector depends only on the orientation of the surface and not on dynamical consideration, $\boldsymbol{n}$ is only a function of S, that is

$$\boldsymbol{n} = \frac{\nabla S}{|\nabla S|} = (-\eta_x\boldsymbol{e}_x + \boldsymbol{e}_y)(1+\eta_x^2)^{-1/2}, \tag{2.3}$$

where $\boldsymbol{e}_x$ is the unit vector along the x-direction and the subscript x refers to the partial derivative with respect to the variable x.

In formulating Maxwell's equations, we suppose that the electroquasistatic approximation [14] is valid for the problem at hand. With this model it is recognized that the relevant rates of change are so small that contributions due to a particular dynamical process are ignorable. The objective in the electrified fluids is concerned with phenomena where electric energy storage much exceeds magnetic energy storage, and where the propagation of electromagnetic waves is fast compared to those of interest to us. Accordingly, the Maxwell's equations reduce to

$$\nabla \wedge \boldsymbol{E} = 0, \tag{2.4}$$

$$\nabla . \varepsilon \boldsymbol{E} = \sigma_f. \tag{2.5}$$

Free surface charges, as addressed by the right-hand side of (2.5), are present due to different electrophysical properties of the two fluids. Due to the presence of free surface charges at the interface, the initial electric displacement is discontinuous by this surface charge density. Therefore, the following relation arizes:

$$\varepsilon^{(1)} E_o^{(1)} - \varepsilon^{(2)} E_o^{(2)} = \sigma_f. \tag{2.6}$$

Since free charges occur only at the interface, in the bulk, (2.5) reduces to

$$\nabla . \varepsilon \boldsymbol{E} = 0. \tag{2.7}$$

In accordance with the validity of the quasistatic approximation, a potential function $\phi(x,y,t)$ may be introduced such that

$$\boldsymbol{E} = -\nabla \phi, \tag{2.8}$$

where the function $\phi(x,y,t)$ satisfies the Laplace equation

$$\nabla^2 \phi = 0, \tag{2.9}$$

and satisfies the following boundary conditions:

(I) $\nabla \phi$ vanishes far from the fluid interface

$$\nabla \phi_j(x, \pm\infty, t) = 0, \qquad j = 1,2. \tag{2.10}$$

(II) At the dividing surface, the potential $\phi(x,y,t)$ must satisfy the conditions:

(i) The jump of the tangential component of the electric field is zero across the interface,

$$\text{i. e.} \qquad \boldsymbol{n} \wedge (\boldsymbol{E}^{(1)} - \boldsymbol{E}^{(2)}) = 0, \qquad y = \eta, \tag{2.11}$$

or

$$\begin{aligned} &\frac{\partial \eta}{\partial x}\left[(E_o^{(1)} - E_o^{(2)}) + \frac{\partial}{\partial y}(\phi^{(1)} - \phi^{(2)})\right] \\ &\quad + \frac{\partial}{\partial x}(\phi^{(1)} - \phi^{(2)}) = 0, \qquad y = \eta. \end{aligned} \tag{2.12}$$

(ii) The tangential component of the stress tensor is continuous across the interface, which leads to

$$\begin{aligned} &\varepsilon^{(1)} E_o^{(1)} \frac{\partial \phi^{(1)}}{\partial x} - \varepsilon^{(2)} E_o^{(2)} \frac{\partial \phi^{(2)}}{\partial x} \\ &\quad + \frac{\partial \eta}{\partial x}\left(\varepsilon^{(1)} E_o^{(1)^2} - \varepsilon^{(2)} E_o^{(2)^2}\right) = 0, \\ &y = \eta. \end{aligned} \tag{2.13}$$

These boundary conditions are prescribed at the interface $y = \eta$. As the interface is deformed, all variables are slightly perturbed from their equilibrium values. Because the interfacial displacement is small, these boundary conditions need to be evaluated at the equilibrium position rather than at the interface. Therefore it is necessary to express all physical quantities in terms of the Maclaurin series about $y = 0$.

The flow through porous media is of considerable interest for petroleum engineers and in geophysical fluid dynamics. Darcy's equation is macroscopic. It describes an incompressible fluid and a viscosity dominated flow. Through the porous medium, it is described by Darcy's law [23]

$$\rho\left[\frac{\partial \boldsymbol{v}}{\partial t} + (\boldsymbol{v}.\nabla)\boldsymbol{v}\right] = -\nabla \pi - \rho g \boldsymbol{e}_y - \mu_o \boldsymbol{v}, \tag{2.14}$$

associated with the incompressibility condition

$$\nabla . \boldsymbol{v} = 0, \tag{2.15}$$

where π is the hydrodynamic pressure, μ_o is Darcy's coefficient which depends on the ratio of the fluid viscosity to the flow permeability, and $\boldsymbol{v} = (u,v)$ is the fluid velocity.

The pure equilibrium configuration yields

$$\pi_o^{(j)}(x,y,t) = -\rho^{(j)} g y + C_o^{(j)}, \tag{2.16}$$

where $C_o^{(j)}$ is the integration constant, which is determined from the balance of the normal stress tensor by

$$C_o^{(2)} - C_o^{(1)} = \frac{1}{2}\left(\varepsilon^{(1)} E_o^{(1)^2} - \varepsilon^{(2)} E_o^{(2)^2}\right). \tag{2.17}$$

We want to develop a method of solution that is not restricted to infinitesimal deformations. Therefore, we were interested in a weak non-linear approach that is based on the idea of using linear solutions under non-linear boundary conditions.

Far from the interface we have

$$\boldsymbol{v}^{(j)}(x,\pm\infty,t)=0. \tag{2.18}$$

In addition, at the interface it is required that both the horizontal and vertical components of the fluid velocity satisfy an equation expressing the assumed material character of the dividing surface. Such an equation may be

$$v^{(j)}=\frac{\partial\eta}{\partial t}+u^{(j)}\frac{\partial\eta}{\partial x}. \tag{2.19}$$

Finally there is the dynamical condition that the normal stress should be continuous across the interface, which gives

$$(\sigma_{yy}^{(1)}-\sigma_{yy}^{(2)})+\eta_x^2(\sigma_{xx}^{(1)}-\sigma_{xx}^{(2)})-2\eta_x(\sigma_{xy}^{(1)}-\sigma_{xy}^{(2)})=-\sigma\eta_{xx}(1+\eta_x^2)^{-1/2},\quad y=\eta \tag{2.20}$$

where the stress tensor σ_{ij} is given by

$$\sigma_{ij}=-(\pi+\frac{1}{2}\varepsilon E^2)\delta_{ij}+\varepsilon E_iE_j, \tag{2.21}$$

where δ_{ij} is Kronecker's delta. The surface tension is denoted by σ. It should be noted that the interface between the two fluids is assumed to be well defined and initially flat. In fact, a sharp interface between the two fluids may not exist. Rather, there is an ill-defined transition region in which the two fluids mix. The width of this transition zone is usually small compared to the characteristic length of the motion. Hence, for the purpose of the mathematical analysis, we shall assume that the two fluids are separated by a sharp interface.

Combining equations (2.20) and (2.21), one finds

$$\begin{aligned}&\pi^{(2)}-\pi^{(1)}+(\rho^{(1)}-\rho^{(2)})g\eta-(1-2\eta_x^2)\\&\cdot\Big[\varepsilon^{(1)}E_0^{(1)}\frac{\partial\phi^{(1)}}{\partial y}-\varepsilon^{(2)}E_0^{(2)}\frac{\partial\phi^{(2)}}{\partial y}\\&\quad+\frac{1}{2}\varepsilon^{(1)}((\frac{\partial\phi^{(1)}}{\partial x})^2-(\frac{\partial\phi^{(1)}}{\partial y})^2)\\&\quad-\frac{1}{2}\varepsilon^{(2)}((\frac{\partial\phi^{(2)}}{\partial x})^2-(\frac{\partial\phi^{(2)}}{\partial y})^2)\Big]+2\eta_x(1-\eta_x^2)\\&\cdot\Big[\varepsilon^{(1)}E_0^{(1)}\frac{\partial\phi^{(1)}}{\partial x}-\varepsilon^{(2)}E_0^{(2)}\frac{\partial\phi^{(2)}}{\partial x}-\varepsilon^{(1)}\frac{\partial\phi^{(1)}}{\partial x}\frac{\partial\phi^{(1)}}{\partial y}\\&\quad+\varepsilon^{(2)}\frac{\partial\phi^{(2)}}{\partial x}\frac{\partial\phi^{(2)}}{\partial y}\Big]=-\sigma\eta_{xx}(1-\frac{3}{2}\eta_x^2),\\&y=\eta.\end{aligned} \tag{2.22}$$

3. Derivation of the Non-linear Characteristic Equation

The analysis of the linear stability theory as presented in Chandrasekhar's book [8] depends on neglecting the non-linear terms in the equations of motion as well as in the boundary conditions. Therefore the resulting dispersion relation contains no non-linear terms. The idea of the weakly non-linear approach slightly differs from the linear approach. To this end, the non-linear problem will contain a linear description with some additional terms that make a correction to the main solution. The weakly non-linear description given here depends on neglecting the non-linear terms from the equations of motion and applying appropriate non-linear boundary conditions. Therefore, the dispersion relation should be extended to include non-linear terms.

To solve the linearization equation for the system under consideration, two-dimensional finite disturbances are introduced into the equations of motion as well as the boundary conditions. In view of a standard Fourier decomposition, the surface deflection (2.2) may be written as

$$\eta=\alpha e^{i(kx-\omega t)}+\text{c.c.}, \tag{3.1}$$

where η denotes the elevation of the interface at the time t, k is the wave number, which is assumed to be real and positive, ω is the growth rate, α an arbitrary constant, which determines the amplitude of the disturbance of the interface and $c.c.$ indicates the complex conjugate of the preceding term. It should be noted that the imaginary part of ω indicates a disturbance which either grows with time (instability) or decays with time (stability), depending on this imaginary part being positive or negative, respectively.

In the light of the shape of the surface elevation, we may similarly assume that the bulk solutions have the form

$$\chi(x,y,t)=\hat{\chi}(y)e^{i(kx-\omega t)}, \tag{3.2}$$

where χ stands for $\boldsymbol{v},\pi$ and ϕ.

By means of the continuity equations (2.15) one eliminates the dependence on the pressure function in the linearized equation of motion (2.14). The resulting equation represents the Laplace equation on the velocity vector, which leads to the following solutions that are compatible with the non-linear boundary condition (2.19):

$$u^{(1)}(x,y,t) = \frac{\eta_t}{i-\eta_x} e^{-ky} \text{ for } y>0, \tag{3.3}$$

$$u^{(2)}(x,y,t) = \frac{-\eta_t}{i+\eta_x} e^{ky} \text{ for } y<0. \tag{3.4}$$

Now, the pressure distributions in the two fluids become

$$\pi^{(1)}(x,y,t) = \frac{i}{k(i-\eta_x)} \left(\rho^{(1)}\eta_{tt} + \mu_o^{(1)}\eta_t\right) e^{-ky} \quad \text{for} \quad y>0, \tag{3.5}$$

$$\pi^{(2)}(x,y,t) = \frac{-i}{k(i+\eta_x)} \left(\rho^{(2)}\eta_{tt} + \mu_o^{(2)}\eta_t\right) e^{ky} \quad \text{for} \quad y<0. \tag{3.6}$$

It should be noted that the equations (3.3), (3.4), (3.5) and (3.6), contain non-linear terms in the elevation parameter η. This non-linearity occurs because of the non-linear boundary conditions. If the non-linear terms are ignored, linear profiles arise which are equivalent to those obtained earlier by Chandrasekhar [8].

For the electric part, the solution of the Laplace equation (2.9) that is compatible with the non-linear boundary conditions (2.12) and (2.13) may be represented in the form

$$\phi^{(1)}(x,y,t) = -\frac{iE_o^{(1)}(E_o^{(1)}-E_o^{(2)})\eta_x e^{-ky}}{k(E_o^{(1)}+E_o^{(2)})(1+i\eta_x)} \quad \text{for} \quad y>0, \tag{3.7}$$

$$\phi^{(2)}(x,y,t) = \frac{iE_o^{(2)}(E_o^{(1)}-E_o^{(2)})\eta_x e^{ky}}{k(E_o^{(1)}+E_o^{(2)})(1-i\eta_x)} \quad \text{for} \quad y<0. \tag{3.8}$$

If the non-linear terms are ignored in the electric potential distributions (3.7) and (3.8), linear profiles arise which are equivalent to those obtained by Melcher [14].

At the boundary between the two fluids, the fluids and the electrical stresses must be balanced. These stresses consist of hydrodynamic pressure, surface tension, velocities and electric potentials. Inserting the solutions (3.3) – (3.8) into the normal stress tensor (2.22), a very complicated non-linear equation for the elevation η arises. Keeping in mind that the elevation η is small, the binomial expansion is convenient. For terms up to the third order of η, one finds

$$\begin{aligned}
&\Big\{-(\rho^{(1)}+\rho^{(2)})\eta_{tt} - (\mu_o^{(1)}+\mu_o^{(2)})\eta_t \\
&\quad + k(\rho^{(1)}-\rho^{(2)})g\eta + k\sigma\eta_{xx} \\
&\quad - ik(\varepsilon^{(1)}E_o^{(1)^2} + \varepsilon^{(2)}E_o^{(2)^2})\eta_x\Big\} \\
&+ \Big\{i(\rho^{(1)}-\rho^{(2)})\eta_t\eta_{tx} + i(\mu_o^{(1)}-\mu_o^{(2)})\eta_t\eta_x \\
&\quad - k(\varepsilon^{(1)}E_o^{(1)^2} - \varepsilon^{(2)}E_o^{(2)^2})\eta_x^2\Big\} \\
&- k\Big\{(\rho^{(1)}-\rho^{(2)})g\eta - \frac{1}{2}\sigma\eta_{xx}\Big\}\eta_x^2 + \cdots = 0.
\end{aligned} \tag{3.9}$$

According to Grimshaw's theory [29], the non-linear terms of the characteristic equation (3.9) consist of two parts: The first part contains the interaction of the second harmonic term. The second one represents the cubic interaction of the primary harmonic term, which is identical with the same term that arises in the Stokes expansion for a plane progressive wave.

Equation (3.9) is more general than that obtained by Raghavan and Marsden [30], Bau [25], Sharma and Spanos [24]. In addition to the non-linear contributions, it includes the electric field influence. It is more convenient to introduce the following transformation:

$$\eta(x,t) = \xi(x,t)e^{-\frac{1}{2}\mu^* t}, \tag{3.10}$$

where $\mu^* = (\mu_o^{(1)}+\mu_o^{(2)})/(\rho^{(1)}+\rho^{(2)})$.

Accordingly, the characteristic non-linear equation (3.9) becomes

$$\begin{aligned}
&\Big\{-\xi_{tt} + \frac{k}{\rho^{(1)}+\rho^{(2)}}\Big[(\rho^{(1)}-\rho^{(2)})g\xi + k\sigma\xi_{xx} \\
&\quad - ik(\varepsilon^{(1)}E_o^{(1)^2}+\varepsilon^{(2)}E_o^{(2)^2})\xi_x\Big] + \frac{1}{4}\mu^{*2}\Big\}e^{-\frac{1}{2}\mu^* t} \\
&+ \frac{i}{\rho^{(1)}+\rho^{(2)}}\Big[(\rho^{(1)}-\rho^{(2)})(\xi_t - \frac{1}{2}\mu^*\xi)(\xi_{tx} - \frac{1}{2}\mu^*\xi) \\
&\quad + (\mu_o^{(1)}-\mu_o^{(2)})(\xi_t\xi_x - \frac{1}{2}\mu^*\xi_x\xi) \\
&\quad + ik(\varepsilon^{(1)}E_o^{(1)^2}+\varepsilon^{(2)}E_o^{(2)^2})\xi_x^2\Big]e^{-\mu^* t} + \frac{k}{\rho^{(1)}+\rho^{(2)}} \\
&\cdot\Big[(\rho^{(1)}-\rho^{(2)})g\xi - \frac{1}{2}\sigma\xi_{xx}\Big]\xi_x^2 e^{-\frac{3}{2}\mu^* t} + \cdots = 0.
\end{aligned} \tag{3.11}$$

On neglecting the higher orders of ξ, the linearized form of equation (3.11) becomes

$$L(\frac{\partial}{\partial t},\frac{\partial}{\partial x})\xi = 0, \qquad (3.12)$$

where L is a linear operator involving the spatial and temporal partial derivatives, which is defined as

$$L\Big(\frac{\partial}{\partial t},\frac{\partial}{\partial x}\Big) \equiv -\frac{\partial^2}{\partial t^2} + \frac{k}{\rho^{(1)}+\rho^{(2)}}\Big[k\sigma\frac{\partial^2}{\partial x^2} - ik(\varepsilon^{(1)}E_{\rm o}^{(1)^2} + \varepsilon^{(2)}E_{\rm o}^{(2)^2})\frac{\partial}{\partial x} + (\rho^{(1)}-\rho^{(2)})g\Big] + \frac{1}{2}\mu^{*2}. \qquad (3.13)$$

First we study (3.12), then we return to (3.11) to incorporate the higher order dispersive effects. Suppose we require a uniform monochromatic wavetrain solution of (3.12) in the form

$$\xi(x,t) = \gamma e^{i(kx-\omega t)} + \text{c.c.}, \qquad (3.14)$$

where γ is an arbitrary constant which determines the amplitude of the interface disturbance.

The existence of harmonic wave trains in a dispersive medium and the correspondence between the wave number and the frequency, lead to several physical consequences. In the linear approximation γ is considered a constant. In the non-linear approach it is treated as a slowly varying function of space and time. Combining (3.14) and (3.12) one obtains

$$L(-i\omega, ik)\xi = 0. \qquad (3.15)$$

from which it follows that

$$D(\omega,k)\gamma = 0, \qquad (3.16)$$

where $D(\omega,k)$ represents the linear dispersion function. As $\gamma \neq 0$, we have

$$\omega^2 = \frac{k}{\rho^{(1)}+\rho^{(2)}}\Big[k^2\sigma - (\rho^{(1)}-\rho^{(2)})g - k(\varepsilon^{(1)}E_{\rm o}^{(1)^2} + \varepsilon^{(2)}E_{\rm o}^{(2)^2})\Big] - \frac{1}{4}\mu^{*2}. \qquad (3.17)$$

It follows that, through the linear stability theory, the surface tension is stabilizing. On the other hand, both the electric field and Darcy's coefficients are destabilizing. It should be noted that (3.17) represents the linear dispersion relation that is satisfied by the values of ω and k. To develop the non-linear effects for the amplitude modulation of progressive waves we have to consider the full non-linear equation (3.11), taking into account the linear dispersion relation (3.17).

In order to investigate the non-linear stability criteria of our system we modulate the problem so that the linear dispersion relation $D(\omega,k)$ represents the slowly modulated wave train. At this stage we may use the expansion procedure obtained by the method of multiple scales [3]. The underlying idea of this method is to make an expansion, representing the solution of the problem as a function of two or more independent variables. Consider a small parameter δ which measures the ratio of a typical wave length or time scale of the modulation. The independent variables x and t may be expanded to introduce alternative independent variables as

$$T_n = \delta^n t, \quad X_n = \delta^n x, \qquad n = 0,1,2. \qquad (3.18)$$

Therefore, defining $T_{\rm o}$ and $X_{\rm o}$ as variables appropriate for fast variation, T_1, X_1, T_2 and X_2 are slow ones. Using similar arguments as given in [12], one obtains the following Ginzburg-Landau equation:

$$i\frac{\partial\gamma}{\partial\tau} + P\frac{\partial^2\gamma}{\partial\xi^2} = Q\gamma^2\bar{\gamma}e^{-\mu^*\delta^{-2}\tau}, \qquad (3.19)$$

where

$$P = -\frac{1}{2}\frac{{\rm d}^2\omega}{{\rm d}k^2} = \frac{-1}{8\omega^3(\rho^{(1)}+\rho^{(2)})}\Big(\Big[3k^2\sigma + :(\varepsilon^{(1)}E_{\rm o}^{(1)^2} + \varepsilon^{(2)}E_{\rm o}^{(2)^2}) - (\rho^{(1)}+\rho^{(2)})g\Big]^2 - 4\omega^2(\rho^{(1)}+\rho^{(2)})(3k\sigma - (\varepsilon^{(1)}E_{\rm o}^{(1)^2} + \varepsilon^{(2)}E_{\rm o}^{(2)^2})\Big),$$

and

$$Q = \frac{-k^2}{2\omega(\rho^{(1)}+\rho^{(2)})} \cdot \Big\{\frac{2}{2k(2k^2\sigma + g(\rho^{(1)}-\rho^{(2)})) + \frac{3}{4}(\rho^{(1)}+\rho^{(2)})\mu^{*2}} \cdot \Big[(\rho^{(1)}-\rho^{(2)})(i\omega+\frac{1}{2}\mu^*)(\omega+\frac{1}{2}\mu^*) + (\mu_{\rm o}^{(1)}-\mu_{\rm o}^{(2)}) \cdot (i\omega+\frac{1}{2}\mu^*) + k^2(\varepsilon^{(1)}E_{\rm o}^{(1)^2} + \varepsilon^{(2)}E_{\rm o}^{(2)^2})\Big]^2 + k[(\rho^{(1)}-\rho^{(2)})g + \frac{1}{2}k^2\sigma]\Big\}.$$

In the light of the transformation

$$\Lambda(\xi,\tau) = \gamma(\xi,\tau)e^{-\frac{1}{2}\mu^*\delta^{-2}\tau}, \tag{3.20}$$

the Ginzburg-Landau equation (3.19) reduces to

$$i\frac{\partial\Lambda}{\partial\tau} + P\frac{\partial^2\Lambda}{\partial\xi^2} + \frac{1}{2}\delta^{-2}\mu^*\Lambda = Q\Lambda^2\bar{\Lambda}. \tag{3.21}$$

Clearly the coefficient P has been constructed depending on the wave frequency ω. Through the marginal state it is real, while the non-linear coefficient Q is complex ($Q = Q_r + iQ_i$). To this end the stability criteria for the Ginzburg-Landau equation (3.20) are presented, hence the following conditions are satisfied [31]:

$$PQ_r > 0 \quad \text{and} \quad Q_i < 0. \tag{3.22}$$

In terms of the electric field intensity, these conditions may be arranged, respectively, as

$$(P_1E_o^{(1)^2} + P_o)(a_2E_o^{(1)^4} + a_1E_o^{(1)^2} + a_o) > 0, \tag{3.23}$$

$$b_2E_o^{(1)^4} + b_1E_o^{(1)^2} + b_o < 0. \tag{3.24}$$

The significance of the coefficients P', a' and b' is clear in this context and is not included here. Therefore the transition curves, separating stable from unstable regions, correspond to

$$P_1E_o^{(1)^2} + P_o = 0, \tag{3.25}$$

$$a_2E_o^{(1)^4} + a_1E_o^{(1)^2} + a_o = 0, \tag{3.26}$$

$$b_2E_o^{(1)^4} + b_1E_o^{(1)^2} + b_o = 0. \tag{3.27}$$

These marginal curves may be obtained by a numerical estimation.

4. Kelvin-Helmholtz Instability

In what follows we wish to consider a generalization of the non-linear KHI for flow in porous media under the influence of a vertical electric field. However, one must addresses some additional complexities that are principally due to the interactions between the electric field and the porous material in the non-linear analysis. Furthermore, a difficulty arises when the two fluids are in relative motion.

In this case, the pure equilibrium configuration yields

$$\pi_o^{(j)} = -\rho^{(j)}gy - \mu_o^{(j)}V_o^{(j)}x + c_o^{(j)}, \; j = 1,2, \tag{4.1}$$

where $c_o^{(j)}$ are the integration constants, and $V_o^{(1)}$ and $V_o^{(2)}$ are the uniform horizontal velocities of the upper and lower fluids, respectively.

To solve the linearized equations of the system under consideration, two-dimensional finite disturbances are introduced to the equations of motion (2.9) and (2.14) and the continuity equation (2.15) as well as the non-linear boundary conditions (2.12), (2.13), (2.18) and (2.19). In the present case, the continuity equation (2.15) allows introducing a stream function $\psi(x,y,t)$, which is defined as

$$\boldsymbol{v} = (V_o - \psi_y)\boldsymbol{e}_x + \psi_x\boldsymbol{e}_y. \tag{4.2}$$

As stated in the previous case, one finds the following solutions:

$$\psi^{(1)}(x,y,t) = -\frac{i(\eta_t + V_o^{(1)}\eta_x)}{k(1+i\eta_x)}e^{-ky} \text{ for } y > 0, \tag{4.3}$$

$$\psi^{(2)}(x,y,t) = -\frac{i(\eta_t + V_o^{(2)}\eta_x)}{k(1-i\eta_x)}e^{ky} \text{ for } y < 0, \tag{4.4}$$

$$\pi^{(1)}(x,y,t) = \frac{1}{k(1+i\eta_x)}\left(\frac{\partial}{\partial t} + V_o^{(1)}\frac{\partial}{\partial x}\right)\Big[\rho^{(1)}\frac{\partial}{\partial t} + \rho^{(1)}V_o^{(1)}\frac{\partial}{\partial x} + \mu_o^{(1)}\Big]\eta e^{-ky}, \; y > 0, \tag{4.5}$$

and

$$\pi^{(2)}(x,y,t) = \frac{-1}{k(1-i\eta_x)}\left(\frac{\partial}{\partial t} + V_o^{(2)}\frac{\partial}{\partial x}\right)\Big[\rho^{(2)}\frac{\partial}{\partial t} + \rho^{(2)}V_o^{(2)}\frac{\partial}{\partial x} + \mu_o^{(2)}\Big]\eta e^{ky}, \; y < 0. \tag{4.6}$$

This problem has been treated particularly for pure viscous fluids in the absence of porous media and an electric field by Chandrasekhar [8]. He showed that the surface tension suppresses completely the KHI for small wavelengths. Sharma and Spanos [24] and Raghaven and Mardsen [30] investigated the additional problem of the KHI for flow in porous media. The problem was discussed for a Darcy-type flow by El-Sayed [32] in porous media under the influence of a tangential electric field to the interface between the two fluids.

In accordance with the procedure explained in the preceding section, one can establish the following non-linear equation governing the surface elevation η in the case of KHI:

$$(\rho^{(1)}+\rho^{(2)})\eta_{tt}+2(\rho^{(1)}V_o^{(1)}+\rho^{(2)}V_o^{(2)})\eta_{xt} + (\rho^{(1)}V_o^{(1)^2}+\rho^{(2)}V_o^{(2)^2})\eta_{xx}+(\mu_o^{(1)}+\mu_o^{(2)})\eta_t + (\mu_o^{(1)}V_o^{(1)}+\mu_o^{(2)}V_o^{(2)})\eta_x+\Big[(\varepsilon^{(1)}E_o^{(1)^2} + \varepsilon^{(2)}E_o^{(2)^2})-k\sigma\Big]\eta_{xx}-kg(\rho^{(1)}-\rho^{(2)})\eta$$
$$-i\Big[(\rho^{(1)}-\rho^{(2)})\eta_t\eta_{xt}+(\rho^{(1)}V_o^{(1)}-\rho^{(2)}V_o^{(2)})\eta_t\eta_{xx} +2(\rho^{(1)}V_o^{(1)^2}-\rho^{(2)}V_o^{(2)^2})\eta_x\eta_{xx} +(\mu_o^{(1)}-\mu_o^{(2)})\eta_t\eta_x-(\mu_o^{(1)}V_o^{(1)}-\mu_o^{(2)}V_o^{(2)})\eta_x^2 +k(\varepsilon^{(1)}E_o^{(1)^2}-\varepsilon^{(2)}E_o^{(2)^2})\eta_{xx}\eta_x\Big]$$
$$-\Big[(\rho^{(1)}+\rho^{(2)})\eta_x^2\eta_{tt}+2(\rho^{(1)}V_o^{(1)}+\rho^{(2)}V_o^{(2)})\eta_t\eta_x\eta_{xx} +(\rho^{(1)}V_o^{(1)^2}+\rho^{(2)}V_o^{(2)^2})\eta_x^2\eta_{xx}+(\mu_o^{(1)}+\mu_o^{(2)})\eta_t\eta_x^2 -[\frac{3}{2}\sigma k-(\varepsilon^{(1)}E_o^{(1)^2}+\varepsilon^{(2)}E_o^{(2)^2})]\eta_{xx}\eta_x^2 +(\mu_o^{(1)}V_o^{(1)}+\mu_o^{(2)}V_o^{(2)})\eta_x^3\Big]+\ldots=0. \quad (4.7)$$

The linearization from the non-linear characteristic (4.7) may be written as

$$L(\frac{\partial}{\partial t},\frac{\partial}{\partial x})\eta=0, \quad (4.8)$$

where L is a linear operator.

It should be noted that the linear dispersion relation (4.8) represents a quadratic equation with complex coefficients in the frequency of the disturbance. This equation has two complex roots. As a limiting case, when $\mu_o^{(j)}\to 0$, the linear dispersion relation reduces to the well known relation found earlier by Chandrasekhar [8] in the absence of the electric field. In order to formulate a harmonic wave train solution of the linear problem, which is required as a basic state for the non-linear state, we may introduce the following transformations:

$$T=t-\frac{1}{\lambda}x \quad \text{and} \quad X=x-\lambda t; \qquad \lambda=\frac{\mu_o^{(1)}V_o^{(1)}+\mu_o^{(2)}V_o^{(2)}}{\mu_o^{(1)}+\mu_o^{(2)}}. \quad (4.9)$$

Consequently, the first partial derivatives should be transformed to

$$\frac{\partial}{\partial t}=\frac{\partial}{\partial T}-\lambda\frac{\partial}{\partial X} \quad \text{and} \quad \frac{\partial}{\partial x}=\frac{\partial}{\partial X}-\frac{1}{\lambda}\frac{\partial}{\partial T}.$$

In the light of these transformations, (4.7) reduces to

$$\Big\{\Big[(\rho^{(1)}+\rho^{(2)})\lambda^2-2(\rho^{(1)}V_o^{(1)}+\rho^{(2)}V_o^{(2)})\lambda + (\rho^{(1)}V_o^{(1)^2}+\rho^{(2)}V_o^{(2)^2})-k\sigma+(\varepsilon^{(1)}E_o^{(1)^2} + \varepsilon^{(2)}E_o^{(2)^2})\Big](\frac{\partial}{\partial T}-\lambda\frac{\partial}{\partial X})^2\eta-kg(\rho^{(1)}-\rho^{(2)})\eta\Big\}$$
$$+i\frac{1}{\lambda}\Big[(\rho^{(1)}-\rho^{(2)})\lambda^2+(\rho^{(1)}V_o^{(1)}-\rho^{(2)}V_o^{(2)})\lambda + (\rho^{(1)}V_o^{(1)^2}-\rho^{(2)}V_o^{(2)^2})+(\varepsilon^{(1)}E_o^{(1)^2}-\varepsilon^{(2)}E_o^{(2)^2}) \cdot(\frac{\partial}{\partial T}-\lambda\frac{\partial}{\partial X})^2\eta-2k^2\lambda\frac{(\mu_o^{(1)^2}V_o^{(1)}-\mu_o^{(2)^2}V_o^{(2)})}{\mu_o^{(1)}+\mu_o^{(2)}} \cdot(\frac{\partial}{\partial T}-\lambda\frac{\partial}{\partial X})\eta\Big](\frac{\partial}{\partial T}-\lambda\frac{\partial}{\partial X})\eta$$
$$-\frac{1}{\lambda^2}\Big[(\rho^{(1)}+\rho^{(2)})\lambda^2 - 2(\rho^{(1)}V_o^{(1)}+\rho^{(2)}V_o^{(2)})\lambda+(\rho^{(1)}V_o^{(1)^2}+\rho^{(2)}V_o^{(2)^2}) + (\varepsilon^{(1)}E_o^{(1)^2}+\varepsilon^{(2)}E_o^{(2)^2})-\frac{3}{2}k\sigma(\frac{\partial}{\partial T}-\lambda\frac{\partial}{\partial X})^2\eta -2(\mu_o^{(1)}+\mu_o^{(2)})\lambda^2(\frac{\partial}{\partial T}-\lambda\frac{\partial}{\partial X})\eta\Big](\frac{\partial}{\partial T}-\lambda\frac{\partial}{\partial X})^2\eta +\cdots=0. \quad (4.10)$$

The existence of the harmonic wave train in a disperssive medium requires

$$\eta=\gamma e^{ikX-i(\omega-k\lambda)T}+\text{c.c.}. \quad (4.11)$$

The correspondence between the wave number k and the frequency ω leads to the linear dispersion relation

$$D(\omega,k)=-\omega^2\Big[(\rho^{(1)}+\rho^{(2)})\lambda^2-2(\rho^{(1)}V_o^{(1)} +\rho^{(2)}V_o^{(2)})+(\rho^{(1)}V_o^{(1)^2}+\rho^{(2)}V_o^{(2)^2})-k\sigma + (\varepsilon^{(1)}E_o^{(1)^2}+\varepsilon^{(2)}E_o^{(2)^2})\Big]-kg(\rho^{(1)}-\rho^{(2)})\lambda^2=0. \quad (4.12)$$

Since our attention is focused on the study of the amplitude modulation of progressive waves, we assume that ω is real. To develop the non-linear effects for the amplitude modulation of progressive waves, we must return to the full non-linear characteristic equation (4.10). Applying the perturbation scheme of multiple scales, one gets the solvability condition, which may be used to derive the following non-linear Ginzburg-Landau equation

$$i\frac{\partial\gamma}{\partial\tau}+P\frac{\partial^2\gamma}{\partial\xi^2}=Q\gamma^2\bar{\gamma}, \tag{4.13}$$

where

$$P=-\frac{\omega\left[\lambda^4+2g\sigma\lambda^2(\rho^{(1)}-\rho^{(2)})\omega^2-3\sigma^2\omega^4\right]}{8k^2g^2(\rho^{(1)}-\rho^{(2)})^2\lambda^4},$$

and

$$Q=\frac{-\omega^2}{2kg(\rho^{(1)}-\rho^{(2)})\lambda^6}\Big\{-2i(\mu_o^{(1)}+\mu_o^{(2)})\lambda^4\omega^2$$
$$+k\omega\lambda^2(g(\rho^{(1)}-\rho^{(2)})\lambda^2-\frac{1}{2}\sigma\omega^2)+\frac{\omega^3}{kg(\rho^{(1)}-\rho^{(2)})}$$
$$\cdot\Big[\omega\Big((\rho^{(1)}-\rho^{(2)})\lambda^2+2(\rho^{(1)}V_o^{(1)}-\rho^{(2)}V_o^{(2)})\lambda$$
$$+(\rho^{(1)}V_o^{(1)^2}+\rho^{(2)}V_o^{(2)^2})+(\varepsilon^{(1)}E_o^{(1)^2}+\varepsilon^{(2)}E_o^{(2)^2})\Big)$$
$$-\frac{2i\lambda k^2(\mu_o^{(1)}V_o^{(1)}-\mu_o^{(2)}V_o^{(2)})}{\mu_o^{(1)}+\mu_o^{(2)}}\Big]^2\Big\}. \tag{4.14}$$

In the light of real values of the frequency ω, the coefficient P is real while the non-linear coefficient Q is complex $(Q=Q_r+iQ_i)$. Thus the stability criteria obtained by Lange and Newell [31] become

$$Q_i<0 \quad \text{and} \quad PQ_r>0. \tag{4.15}$$

Otherwiser the system becomes unstable.

Conditions (4.14) may be arranged in terms of the electric field intensity as

$$Q_i=\frac{b_1E^*+b_o}{(\mu_o^{(1)}+\mu_o^{(2)})(E^*+c_o)}<0, \tag{4.16}$$

and

$$PQ_r=\frac{1}{4\lambda^2(\mu_o^{(1)}+\mu_o^{(2)})(E^*+c_o)^4}(E^*-c_1)(E^*+c_2)$$
$$\cdot(a_2E^{*2}+a_1E^*+a_o)>0, \tag{4.17}$$

where $E_o^{(j)^2}=E^*E^{(j)2}$. The significance of the a's, b's and c's is clear from the context. Therefore, the transition curves become

$$b_1E^*+b_o=0, \tag{4.18}$$

$$E^*-c_1=0, \tag{4.19}$$

$$E^*+c_o=0 \tag{4.20}$$

$$E^*+c_2=0, \tag{4.21}$$

$$a_2E^{*2}+a_1E^*+a_o=0. \tag{4.22}$$

The stability picture may be discussed by drawing these curves, where the marginal curves may be born out by numerical estimation. Before dealing with these calculations, it is convenient to introduce an appropriate dimensionless form. This can be done in a number of ways, depending primarily on the choice of the characteristic length. Consider the following dimensionless forms the characteristic length g/ω^2, the characteristic time $1/\omega$ and the characteristic mass σ/ω^2. The other dimensionless quantities are given by

$$k=\hat{k}\frac{\omega^2}{g},\quad \rho^{(j)}=\hat{\rho}^{(j)}\frac{\sigma\omega^4}{g^3},\quad \mu_o^{(j)}=\hat{\mu}_o^{(j)}\frac{\sigma\omega^3}{g^2},$$

$$V_o^{(j)}=\hat{V}_o^{(j)}\frac{g}{\omega} \quad \text{and} \quad E_o^{(j)}=\hat{E}_o^{(j)}\frac{\sigma\omega^2}{\varepsilon^{(2)}g}. \tag{4.23}$$

The superposed ˆ refers to dimensionless quantities will be omitted for simplicity.

5. Numerical Estimation for Stability Configuration

In this section the results of numerical calculations on the stability of surface waves propagating through an interface between two superposed dielectric fluids in porous media are communicated. To this purpose we have calculated the transition curves (3.25) – (3.27) under the stability conditions (3.21). The calculations also include the stability criterion (4.15) by plotting the marginal curves (4.18) – (4.22). In all graphs, the electric field intensity $E_o^{(1)^2}$ is drawn versus the wave number k. S stands for the stable region and U for the unstable one.

In Fig. 1, the variation of the electric field intensity $E_o^{(1)^2}$ is plotted versus the wave number k. The calculations are made for the transition curves (3.25) – (3.27). We collected some variations for Darcy's coefficient $\mu_o^{(1)}$ with $\mu_o^{(2)}$ fixed at unity. The system is

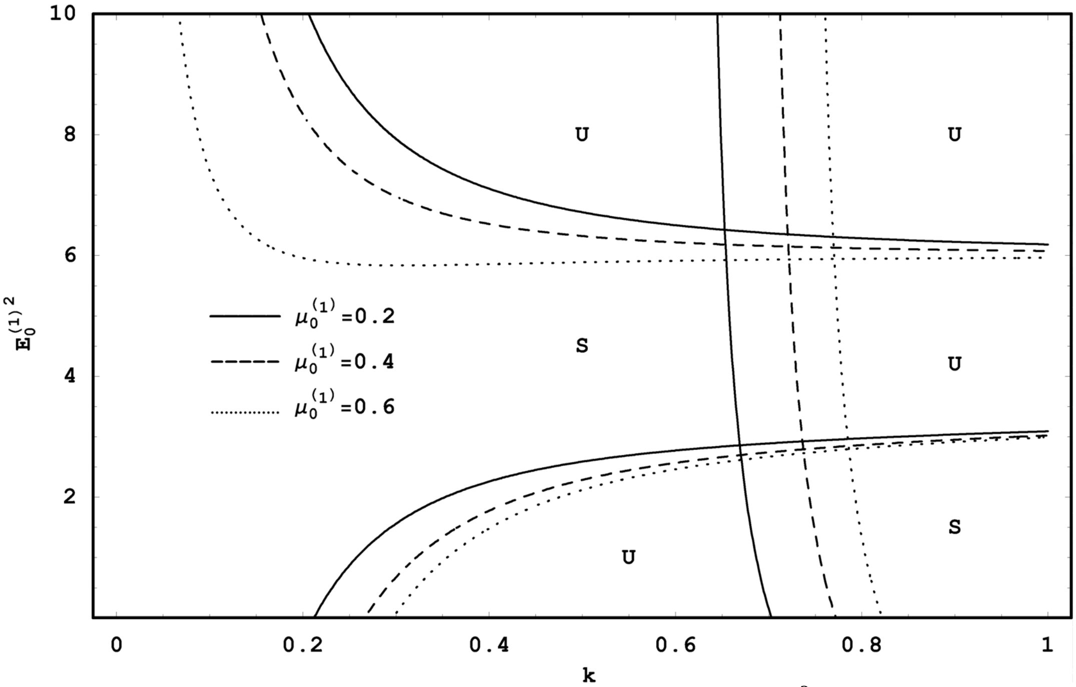

Fig. 1. Stability diagram (S = stable, U = unstable) of the electric field intensity $E_0^{(1)^2}$ versus the wave number k. The calculation is made for a system with $\rho^{(1)} = 0.5$, $\rho^{(2)} = 1$, $\mu_0^{(2)} = 1$, $E_0^{(2)} = 3$ and $\varepsilon = 1.5$ for three values of $\mu_0^{(1)}$. The graph is based on the stability criteria (3.22).

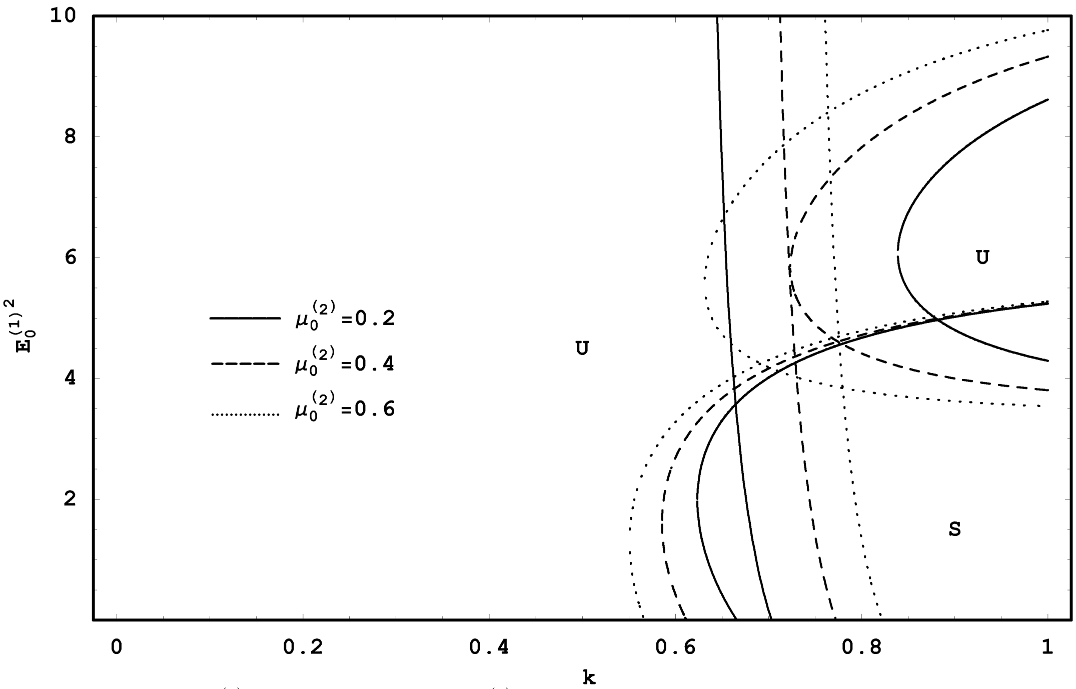

Fig. 2. As in Fig. 1, except $\mu_0^{(1)} = 1$ for three values of $\mu_0^{(2)}$.

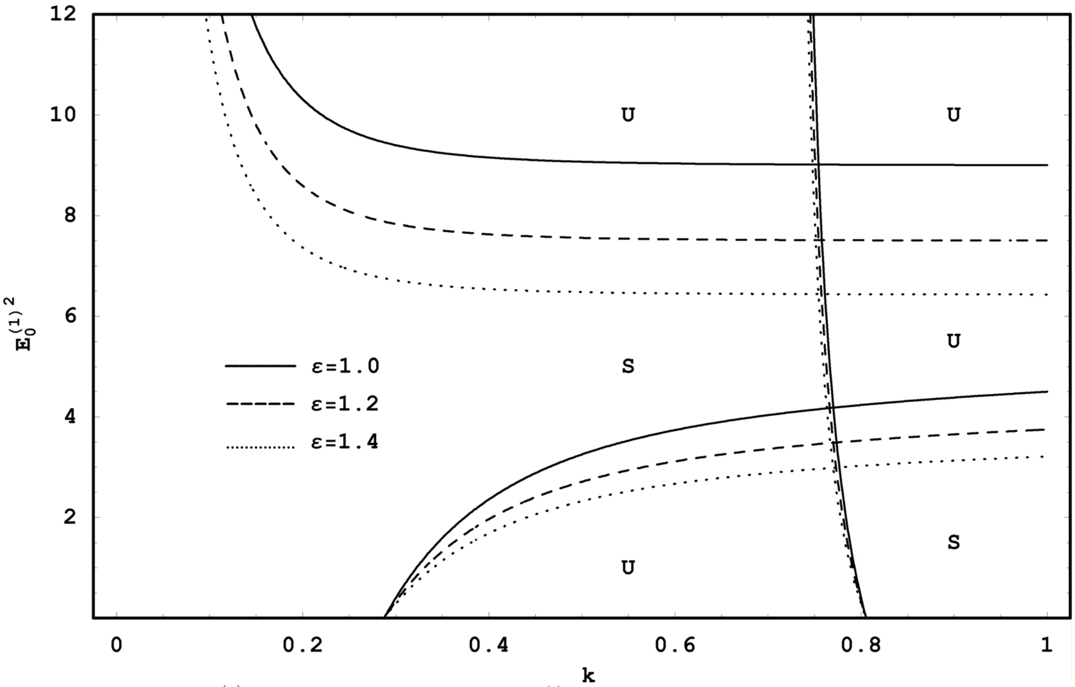

Fig. 3. As in Fig. 1, except $\mu_0^{(1)} = 0.5$ for three values of $\varepsilon = \frac{\varepsilon^{(1)}}{\varepsilon^{(2)}}$.

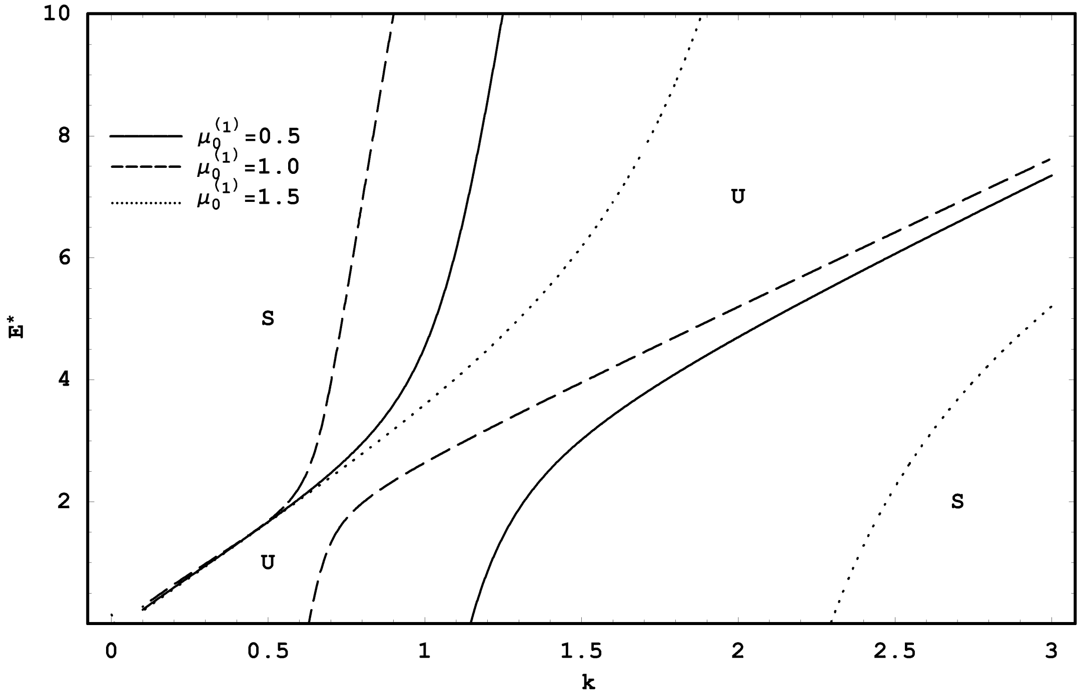

Fig. 4. For streaming fluids in porous media, the stability criteria (4.14) can be represented by stability diagram in the E^* *vs.* k-plane. In Fig. 4 the calculation is made for a system of dimensionless form with $\rho^{(1)} = 0.8$, $\rho^{(2)} = 1$, $V_0^{(1)} = 1$, $V_0^{(2)} = 0.7$, $\mu_0^{(2)} = 1$, $\varepsilon = 0.1$, and $\hat{E}_0^{(1)} = \hat{E}_0^{(2)} = 0.5$ for three values of $\mu_0^{(1)}$.

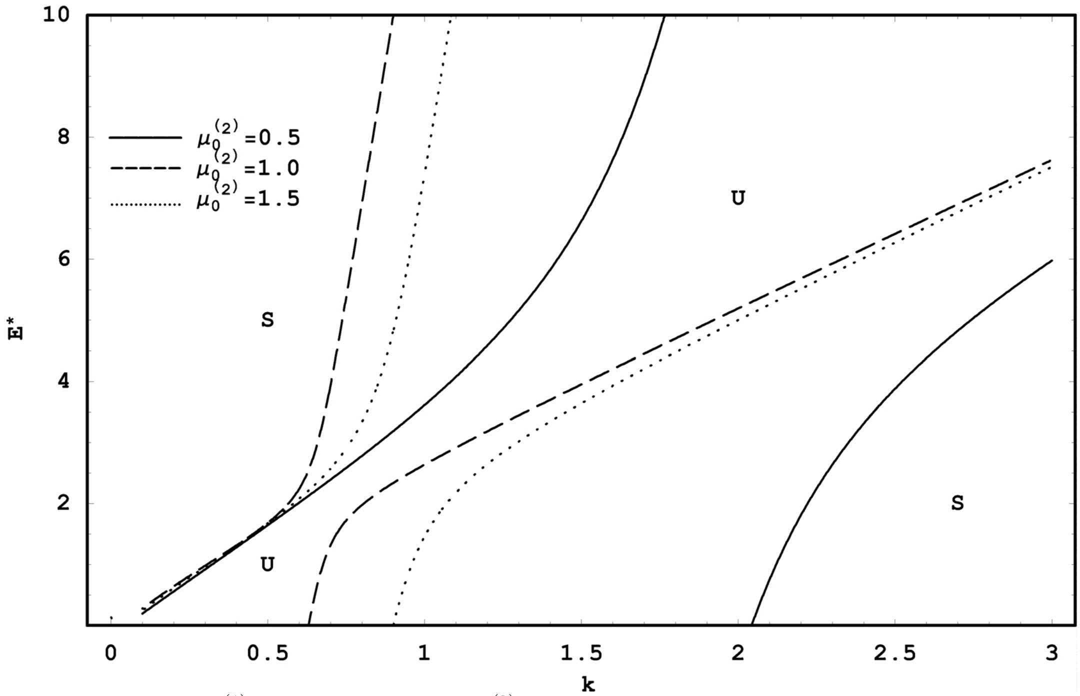

Fig. 5. As in Fig. 4, except $\mu_0^{(1)} = 1$ for three values of $\mu_0^{(2)}$.

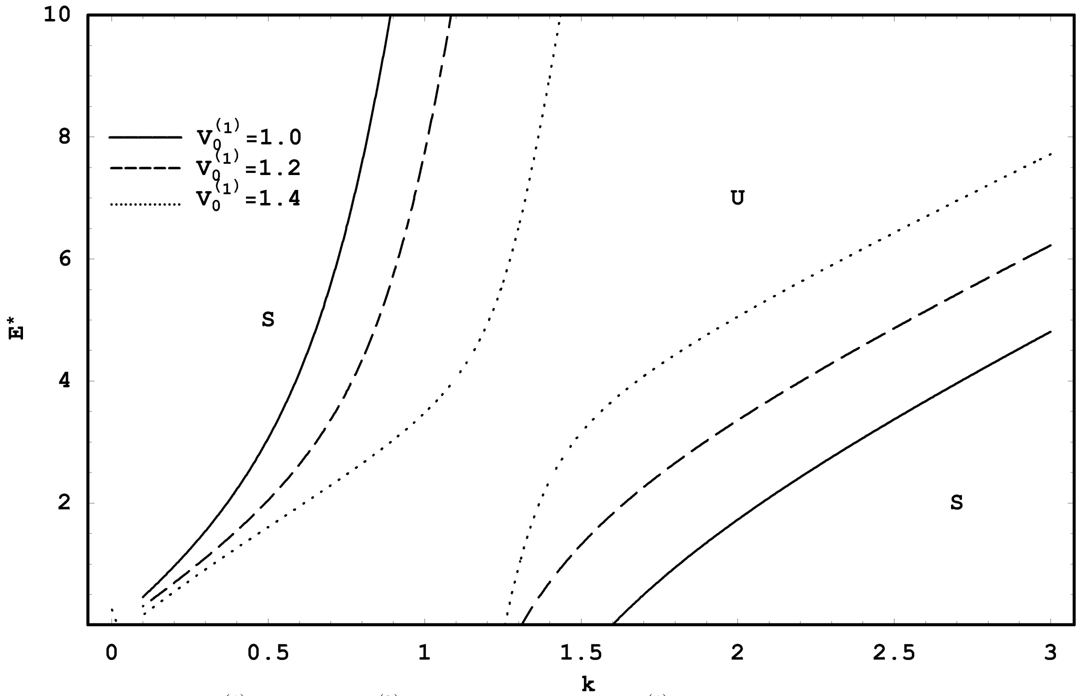

Fig. 6. As in Fig. 4, except $\mu_0^{(1)} = 0.6$, and $V_0^{(2)} = 1$ for three values of $V_0^{(1)}$.

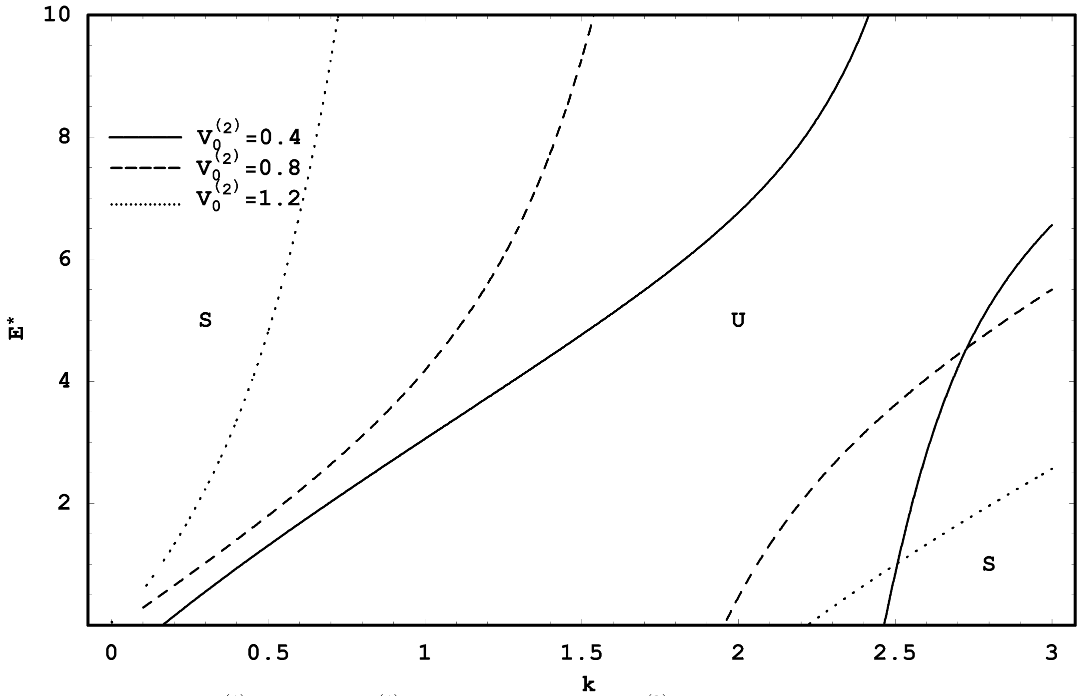

Fig. 7. As in Fig. 4, except $\mu_0^{(1)} = 0.6$, and $V_0^{(1)} = 1$ for three values of $V_0^{(2)}$.

statically stable $(\rho^{(1)} < \rho^{(2)})$. The plane $({E_0^{(1)}}^2 - k)$ is partitioned into stable and unstable regions. It is observed that the vertical electric field is both stabilizing and destabilizing. This shows that the vertical electric field, in the presence of surface charges, plays a dual role on the stability. It is also apparent that the stable regions have decreased as Darcy's parameter $\mu_0^{(1)}$ is increased. This shows the destabilizing influence for the porous medium. The same results have been observed when Darcy's coefficient $\mu_0^{(1)}$ is kept fixed at unity, while $\mu_0^{(2)}$ has some consequence values. This case is shown in Figure 2. The investigation for the influence of the dielectric ratio ε is the subject of Figure 3. The calculations are made for the same system as in Fig. 1, except that $\mu_0^{(1)} = 0.5$. The inspection of this graph reveals that there is a destabilizing influence due to a small increase of ε. Figure 4 shows the influence of Darcy's coefficient $\mu_0^{(1)}$ when the two fluids are in relative motion through the porous media by making numerical calculations for the stability criterion (4.14). This is done by drawing for three values of $\mu_0^{(1)}$ the transition curves (4.17)–(4.21) for a system having $\rho^{(1)} = 0.8$, $\rho^{(2)} = 1$, $V_0^{(1)} = 1$, $V_0^{(2)} = 0.7$, $\mu_0^{(2)} = 1$, $\varepsilon = 0.1$ and $E_0^{(1)} = E_0^{(2)} = 0.5$. The plane E^* *vs.* k is partitioned into stable and unstable regions. The first stable region corresponds to very small wave numbers k. In this region E^* increases when k increases. This shows the stabilizing role for the electric field. The second stable region corresponds to larger values of k. These stable region decreases when the electric field increases. This shows the destabilizing influence of the electric field. We have chosen three schemes of Darcy's parameter $\mu_0^{(1)}$ in this graph. When $\mu_0^{(1)}$ is increased from 0.5 to unity, the first stable region has decreased in width, while the second one has increased. Continuing with increasing $\mu_0^{(1)}$ up to 1.5, the width of the first stable region has increased, while that of the second region has decreases. This behavior is known as a dual role of the stability criteria. The same results are observed in Fig. 5 when increasing $\mu_0^{(2)}$ with fixed $\mu_0^{(1)}$ at unity. The influence of streaming is the subject of Figs. 6 and 7. Three consequence values for the velocity $V_0^{(1)}$ are considered in Fig. 6, with fixed $V_0^{(2)}$ at unity. The inspection of the graph reveals that both the width of the two stable regions has increased as $V_0^{(1)}$ is increased. This shows a stabilizing influence for the increase of the velocity of the upper fluid. When the velocity $V_0^{(1)}$ is switched off and $V_0^{(2)}$ is switched on, different roles may be catched. A destabilizing effect is observed in the first stable region of Fig. 7 and a dual role in the second stable region is found.

6. Conclusions

In the present work, a weakly non-linear calculation is performed in order to investiage a surface waves instability between two superposed streaming dielectric fluids in porous media. The work includes both the RTI and the KHI, considering the interaction of surface tension and a vertical electric field. The interface is assumed to carry surface charges. One pupose was to find out how the vertical electric field governs the non-linear stability of the dielectric interface between the two streaming porous fluids.

The analysis of the linear stability theory presented in Chandrasekhar book [8] for non-porous media, depends on neglecting the non-linear terms from the equations of motion as well as from the boundary conditions and then impose a dispersion in a linear form. The idea of the weakly non-linear description is small departure from the linear point of view. To this end, the non-linear problem contain the linear description with some additional terms representing a correction to the main solution. The weakly non-linear description given here depends on neglecting the non-linear terms from the equations of motion and applying the appropriate boundary conditions without dropping the non-linear terms. At this stage, the dispersion relation has to include non-linear terms.

The present boundary-value problem leads to a non-linear characteristic equation. For the surface elevation this non-linear characteristic equation has a complex nature. The non-linearity is maintained up to the third-order. Owing to formulate a harmonic wave train solution for the linear problem, which is the requirement as a basic state for the non-linear approach, two different transformations are added, one for the case of the RTI and the other for the case of the KHI. In both cases, the linear stability is discussed. The method of multiple scales is adopted with the aid of Taylor expansion to derive the Ginzburg-Landau equation. This equation describes the elevation of the wave train up to cubic order. A convenient choice of a nondimensional form relaxes the complexity of the stability criteria, consequently these criteria have easier form. These stability criteria are discussed in the light of a real frequency of the surface waves. Finally, stability picture is investigated where the electric field intensity is graphed versus the wave number. Through the numerical estimations, the following conclusion are drawn.

In the light of the linear stability theory, it is found that the surface tension plays a stabilizing influence while both the electric field intensity and the Darcy's coefficients have a destabilizing influence. The relative streaming velocity plays a destabilizing role. According to the non-linear approach, it is found that the electric field intensity as well as the streaming have played a dual role on the stability criteria. In the case of RTI, we see that the Darcy's coefficients played a destabilizing influence as happened in the linear analysis. In the case of the KHI, these coefficients have a dual role on the stability criteria.

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